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DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

FINITE ELEMENT ANALYSIS OF AERODYNAMIC HEATING IN THREE-DIMENSIONAL VISCOUS HIGH SPEED COMPRESSIBLE FLOW: AN ASSESSMENT

By

Ken Morgan

and

Earl A. Thornton, Principal Investigator

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia

Under
Research Grant NSG 1321
A.R. Wieting, Technical Monitor
Loads and Aeroelasticity Division

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PREFACE

The review was conducted during a four week visit by Dr. Ken Morgan, University College of Swansea, University of Wales, U.K., to the NASA/Langley Research Center in early summer of 1982. This assessment is based primarily upon his and the principal investigator's knowledge of current finite element literature. In addition, several important technical discussions were held with NASA researchers during Professor Morgan's visit, and the authors would like to express their appreciation for these helpful discussions.

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FINITE ELEMENT ANALYSIS OF AERODYNAMIC HEATING IN THREE-DIMENSIONAL VISCOUS HIGH SPEED COMPRESSIBLE FLOW: AN ASSESSMENT

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Ken Morgan[1] and Earl A. Thornton[2]

SUMMARY

The current capability of the finite element method for solving problems of viscous flow is reviewed. Much work has been directed to the simulation of incompressible flows and the relevant features are described. The methods available for, and the problems associated with, the finite element solution of high speed viscous compressible flows are analyzed. A plan for developing finite element research in this area with experimental support is presented.

INTRODUCTION

Thermal stresses and deformations induced by aerodynamic heating on advanced space transportation vehicles are important concerns in structural design. Nonuniform heating may have a significant effect on the performance of the structures, and effective analytical methods for predicting the structural response are required. For the past few years, the principal investigator has been working closely with the NASA/Langley Research Center in developing new finite element methodology for thermal-structural analysis. The finite element method has excellent capabilities for stress analysis of complex structures, but its capabilities for heat transfer and flow analysis are less well-developed. Some recent progress has been made in development of finite element methodology for heat transfer, although much remains to be done before the method is developed to its full potential.

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For aerodynamic flow analysis the method is still relatively undeveloped in comparison to the mature state of the finite difference method in Computational Fluid Dynamics (CFD). The finite element method is not competitive with popular CFD finite difference techniques such as MacCormack's algorithm (refs. 1-3). Yet the finite element method may offer significant computational advantages in comparison to existing CFD methods and deserves further investigation. This report presents a review of recent finite element progress in viscous flow analysis with recommendations for the important research required to extend the method to determine the aerodynamic heating on space transportation vehicles in three-dimensional viscous compressible flows.

VISCOUS INCOMPRESSIBLE FLOW SIMULATION BY THE FINITE ELEMENT METHOD

Two-Dimensional Studies

Consideration will be restricted to those analyses which have utilized the primitive variable formulation in which the basic unknowns are the velocity components and the pressure. The powerful stream-function/vorticity methods will not be considered as this approach is not directly applicable to the analysis of three-dimensional problems which is our ultimate aim.

The initial work of Taylor and Hood (ref. 4) used the steady state equations with an 8-noded element for the velocity field. Numerical experimentation soon indicated serious problems when the pressure was approximated in a similar manner and they were led to the use of 4-noded bilinear elements for the pressure. The presence of the convective terms in the momentum equations leads to a non-symmetric non-linear matrix system and Hood (ref.5) developed a non-symmetric frontal solver to obtain the solution directly at each stage of a standard iteration process. This direct approach to the solution has been adopted in the majority of the subsequent work in this area. Other element types have been implemented (ref. 6), but generally the restriction to a mixed interpolation for velocity and pressure applies.

If the pressure is not of direct interest, it can be effectively removed from the analysis (with a consequent reduction in computer core and

cost) by adopting the penalty function formulation (ref. 7). The pressure, p, is approximated by

$$p = -\lambda \operatorname{div} \underline{u} \tag{1}$$

where $\underline{u}$ is the velocity vector and λ is a large positive number termed the penalty parameter. The standard incompressibility constraint is then replaced by equation (1) and the pressure can be removed from the equation system. This approach works well in practice and the convergence of the solution to the incompressible solution $\lambda \to \infty$ is now the subject of theoretical study (ref. 8).

In the solution of transient problems, the usual approach adopted has been a straightforward implicit algorithm with variants of the Newton-Raphson method to speed convergence (ref. 6). A notable exception is the work of Donea et al. (ref. 9) who produced a finite element explicit fractional step method for time-dependent problems that was based on an algorithm originally developed in the context of finite differences by Chorin (ref. 10). At each time step an auxiliary velocity field is first computed, which accounts for all contributions to the momentum equations, except those arising from the pressure. The pressure field is then obtained by solving the Galerkin equivalent of a Laplace type equation, and the time step is completed by adding pressure contributions to the auxiliary velocities to ensure maintenance of incompressibility.

Over the past few years, the literature in this area has been full of discussions about the need for upwinding. This is a stabilization technique which has been used by many investigators in problems where the convective terms dominate (ref. 11). The basic idea is to compensate for numerical instability due to convection by adding a certain amount of numerical dissipation, Figure 1. It is now generally agreed that this added dissipation should be anisotropic with a component only along the velocity vector (ref. 12). Some researchers, notably Gresho et al. (ref. 13), have argued against the use of upwinding techniques, preferring instead mesh refinement in "problem" areas of the flow field. The streamline upwinding method does

involve a free parameter whose value may be exactly determined for steady linear problems in one dimension but generally is to be selected so as to maximize the accuracy. In a recent paper, Donea (ref. 14) claims to have overcome some of the problems associated with the analysis of transient convective transport processes, and he identifies the source of the trouble for hyperbolic problems to be the accuracy of the difference approximation to the time derivative term. He shows that a specific cure has to be devised for each time-integration method and the results obtained for simple problems do appear to be an improvement over those produced by conventional approaches (eg. Figure 2). A detailed assessment of the impact of the technique to more complicated problems is required, however, before specific conclusions may be drawn.

Turbulence effects are also being included in finite element models, and steady turbulent flow has been analyzed successfully using both algebraic (ref. 15) and more sophisticated (ref. 16) turbulent viscosity models.

Three-Dimensional Studies

This is a new area for the finite element method, and it is interesting to notice how it is developing. The major contribution so far has been made by the group of Gresho at Lawrence Livermore Laboratory (refs. 17-19) which is now reporting the solution of problems involving 6400 elements with approximately 45,000 equations. These would appear to be the largest finite element flow computations ever undertaken. To perform these computations efficiently on a CRAY computer, Gresho has left the implicit formulation, which he favoured in two-dimensions, and has adopted an explicit scheme of the Chorin/Donea type and with the simplest elements. He is also investigating the effects of using a highly vectorizable one point quadrature to evaluate the Galerkin integrals instead of the standard 2×2×2 Gauss point distribution which is not so vectorizable. Initial results showed no significant variation in accuracy between the two approaches, but a substantial saving in computer cost is achieved from 7 sec CPU and 13 sec I/O per time step to 0.3 sec CPU and 1.3 sec I/O per time step. The work of this group has indicated, as might be expected, that major problems with computer

storage and execution time can be encountered in the simulation of complex three-dimensional flows, but they have also shown ways in which these problems may be overcome and high quality results can be produced.

Reddy (ref. 20) is also attempting three-dimensional flow calculations and is employing a three-dimensional form of the penalty method. He reports calculations for natural convection in a cubical box, but admits that his computational facilities are such that he is unable to refine the mesh to investigate the accuracy of the solution.

VISCOUS COMPRESSIBLE FLOW SIMULATION BY THE FINITE ELEMENT METHOD

The most significant contribution in this area is the work of Baker (refs. 21-23) who has developed an approach for analyzing three-dimensional viscous compressible flows. He uses a "dissipative" finite element model which, when solving

$$L(\phi) = 0 \qquad \text{in } \Omega \tag{2}$$

replaces the classical weighted residual statement

$$\int_{\Omega} W_i L(\phi)\, d\Omega = 0 \tag{3}$$

by

$$\int_{\Omega} W_i L(\phi)\, d\Omega + \beta \int_{\Omega} W_i\, \nabla\big(L(\phi)\big)\, d\Omega = 0 \tag{4}$$

By performing the usual stability analysis on a linearized one-dimensional equation, Baker is able to show that with appropriate choice of β the resulting linear finite element algorithm is more accurate than an equivalent finite difference form in which the dissipation takes the form of a standard artificial viscosity. The region of interest is initially mapped into the unit cube, and the Jacobian of the transformation is interpolated

over each element in terms of element nodal values. The quantities $\underset{\sim}{J}_e^{-1}$ and det $\underset{\sim}{J}_e$ are stored for each node of each element, and the need for numerical integration during the running of the program is then removed by performing a number of calculations beforehand, and storing the relevant "hypermatrix" information. The final non-linear matrix equation is solved by a direct Newton-Raphson procedure, invoking the tensor matrix product method to significantly reduce the core and CPU requirement. This approach corresponds to the method of approximate factorization which is widely used with the finite difference method and consists of replacing the left-hand side matrix $\underline{A}$ in the Newton-Raphson procedure by

$$\underline{A} \cong \underline{A}_\xi \otimes \underline{A}_\eta \otimes \underline{A}_\zeta \tag{5}$$

where, for example, $\underline{A}_\xi$ is constructed in the same manner as $\underline{A}$ but considering only one dimensional interpolation and differentiation in the transformed direction ξ.

The model has been "tuned" by examining its performance on the one-dimensional shock tube problem, but only computations with the parabolized form of the equations appear to have been reported in three dimensions.

The work of Cooke (ref. 24) is interesting, for although it is concerned with two-dimensional applications, it compares critically the performance of both finite difference and finite element algorithms. The computer time required by the finite element method is shown to be much larger than that required for a finite difference solution and this is identified to be due to the finite element processes of numerical quadrature, assembly and solution. As has already been mentioned, the finite element practitioners who are currently working with three-dimensional configurations are aware of these restrictions and are developing techniques designed at reducing the CPU time penalty of the finite element method. Other criticisms of the finite element method, such as those concerned with the ability to include upwinding, have been removed by recent finite element developments described in the preceding sections. Cooke does emphasize that accurate results appear to be the rule rather than the exception with the finite element method, and that the variable grid capability is the method's greatest asset.

RECENT TRENDS IN SOLVING FINITE ELEMENT SYSTEM EQUATIONS

In addition to the improvements in finite element modelling work already described, basic algorithmic development studies are currently being carried out to improve the competitiveness of the finite element method. This work is aimed at providing economic solution techniques for the system equations. Notable here is the work of Hughes et al. (ref. 25) who propose an element-by-element solution scheme which avoids the assembly process by working solely with the element equations. This approach appears attractive but it needs to be more thoroughly tested. On an operations count basis (see Fig. 3) it should certainly prove advantageous for 3D problems involving a single degree of freedom per node, but will it prove as competitive for systems of interest where the element matrix is typically 40×40, and is it a vectorizable process? Sample results of applying the method to a simple problem are shown in Figure 4.

An alternative approach is suggested by the work of Park (ref. 26) in which the actual equation system is taken to be of the form

$$(\underline{M} + \Delta t\, \underline{K})\, \underline{\phi}^{n} = \underline{f} \qquad (6)$$

where n denotes the time level. Matrix inversion can be avoided if equation (6) is replaced by

$$\underline{M}\, (\underline{I} + \Delta t\, \underline{M}^{-1}\, \underline{L})\, (\underline{I} + \Delta t\, \underline{M}^{-1}\, \underline{U})\, \underline{\phi}^{n} = \underline{f} \qquad (7)$$

where

$$\underline{K} = \underline{L} + \underline{U},\ \underline{L}^{T} = \underline{U} \qquad (8)$$

and $\underline{L}$ and $\underline{U}$ are lower and upper triangular matrices, respectively. Equations (6) and (7) agree to $O(\Delta t^2)$ but it was found that, although stability could often be guaranteed, unacceptable errors are produced, even for intermediate values of Δt. Attempts at using a non-symmetric splitting (i.e. $\underline{L}^T \neq \underline{U}$) in equation (8) were more successful, but the production of

a general method for performing this splitting was not forthcoming. Park, therefore, retained the symmetric split and replaced equation (6) by

$$\left(\underline{M} + \Delta t^2 \underline{m} + \Delta t(\underline{K} + \Delta t \underline{k})\right) \underline{\phi}^n = \underline{f} \tag{9}$$

where $\underline{m}$ and $\underline{k}$ are diagonal matrices yet to be determined. The equation

$$(\underline{M} + \Delta t^2 \underline{m}) \left\{ (\underline{I} + \Delta t\ \hat{\underline{L}}) \ (\underline{I} + \Delta t\ \hat{\underline{U}}) \right\} \underline{\phi}^n = \underline{f} \tag{10}$$

is used instead of equation (7) where now

$$\hat{\underline{L}} = (\underline{M} + \Delta t^2 \underline{m})^{-1} \ (\underline{L} + \frac{\Delta t}{2} \underline{k})$$

$$\hat{\underline{U}} = (\underline{M} + \Delta t^2 \underline{m})^{-1} \ (\underline{U} + \frac{\Delta t}{2} \underline{k}) \tag{11}$$

and it follows that equation (10) reduces to equation (6) provided that

$$\left\{ \underline{m} + \underline{k} + (\underline{L} + \frac{\Delta t}{2} \underline{k}) \ (\underline{M} + \Delta t^2 \underline{m})^{-1} \ (\underline{U} + \frac{\Delta t}{2} \underline{k}) \right\} \underline{\phi}^n = 0. \tag{12}$$

It is not possible to apply equation (12) at level n as required since $\underline{\phi}^n$ is unknown and, therefore, $\underline{k}$ and $\underline{m}$ are determined by applying equation (12) to the solution at level $n-1$. This approach has been shown to remove the poor accuracy associated with the first method for some simple structural dynamics examples, but no evidence is available which supports its adoption for more complicated problems.

In fluid flow modelling it is frequently essential to use a fine mesh in the vicinity of solid boundaries, to ensure adequate representation of the boundary layer, whereas a much coarser mesh may be used in the flow interior. The stability criterion associated with the fine mesh may rule out the use of an explicit method, whereas the explicit stability limit for the coarse mesh alone could be acceptable. The solution in this case may be the use of a mixed implicit-explicit technique in which the elements forming the fine mesh are treated implicitly while those in the coarse mesh are treated explicitly (ref. 27). As an example, Figure 5 shows a finite element mesh with 3 explicit elements and 2 implicit elements and the structure

of the resulting matrix which must be inverted assuming 2 unknowns per node. In the solution process the zero entries outside the profile need not be stored or operated upon. A finite difference equivalent of this type of approach has been investigated by Shang (ref. 28). He found an order of magnitude increase in speed over the explicit method could be achieved, but numerical oscillations were observed with some coarse grid configurations in 3D.

THE DEVELOPMENT OF A FINITE ELEMENT COMPUTER PROGRAM FOR ANALYSIS OF AERODYNAMIC HEATING IN THREE-DIMENSIONAL HIGH SPEED COMPRESSIBLE VISCOUS FLOW

It is clear at the outset that any computer program for the local analysis of three-dimensional flows will require sophisticated pre- and post-processing software for displaying the initial configuration, boundary conditions and the computed results in a convenient fashion. It is not proposed, at least initially, to spend a large amount of time in developing such software but to utilize and develop existing and proposed facilities available at NASA/Langley, e.g. the pre-processing might be accomplished via a variant of PATRAN-G while the post-processing can utilize the software currently in use with, and under development for, 3D finite difference codes. The main effort can then be directed at producing a finite element computer program for solving the equations of high speed compressible flow in, eventually, three dimensions. As has been shown by the work of Gresho (refs. 17-19), the program will only be capable of handling the size of problem envisaged (of the order of 10^5 unknowns), provided it is highly vectorized so as to make full use of the vector processing speed of the Langley Cyber 203 (refs. 29-31). The optimum finite element solution algorithm is by no means apparent at this stage and parallel algorithm and technique investigation will be required before large scale problems can be attempted. The stages of development envisaged are shown in Figure 6. The 2D code, which will be produced initially, will be written quite generally as far as the number of dimensions is concerned, so that the transfer to 3D at the end of Stage 3 should prove straightforward. It is envisaged that an essential feature of the development will be the comparison of numerical results with experimental observations. Preliminary discussions have taken

place with regard to possible experiments that might be useful for code validation in the initial stages of the development. In the later stages, code validation will become more critical and experimental support essential. These later experiments would have to be detailed during Stage 3, by which time any uncertainties and numerical problems needing further investigation would have become apparent.

CONCLUDING REMARKS

This report reviews the current status of viscous flow modelling by the finite element technique. It is apparent that very little published finite element work is available in the area of high speed compressible viscous flow and that this is a challenging field for the finite element method. A review has also been made of some of the techniques which are currently under investigation for the solution of finite element system equations. A proposed program of work leading to the production of a finite element simulator for local analysis of 3D high speed viscous compressible flow, governed by the full Navier-Stokes equations, has been outlined. Experimental support has been identified as an essential feature of the simulator validation process.

APPENDIX

Using tensor notation and the summation convention, the equations of compressible viscous flow can be written as

Continuity
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0$$

Momentum
$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(u_j \rho u_i + p\, \delta_{ij} - \sigma_{ij}) = 0$$

Energy
$$\frac{\partial}{\partial t}(\rho e) + \frac{\partial}{\partial x_j}\left(u_j \rho e + u_j p - \sigma_{kj} u_k - k \frac{\partial T}{\partial x_j}\right) = 0$$

Equation of state (perfect gas)
$$e = C_v T \qquad p = \rho R T$$

Stress/rate of strain law
$$\sigma_{ij} = \mu(T) \cdot \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] - \frac{\mu(T)}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij}$$

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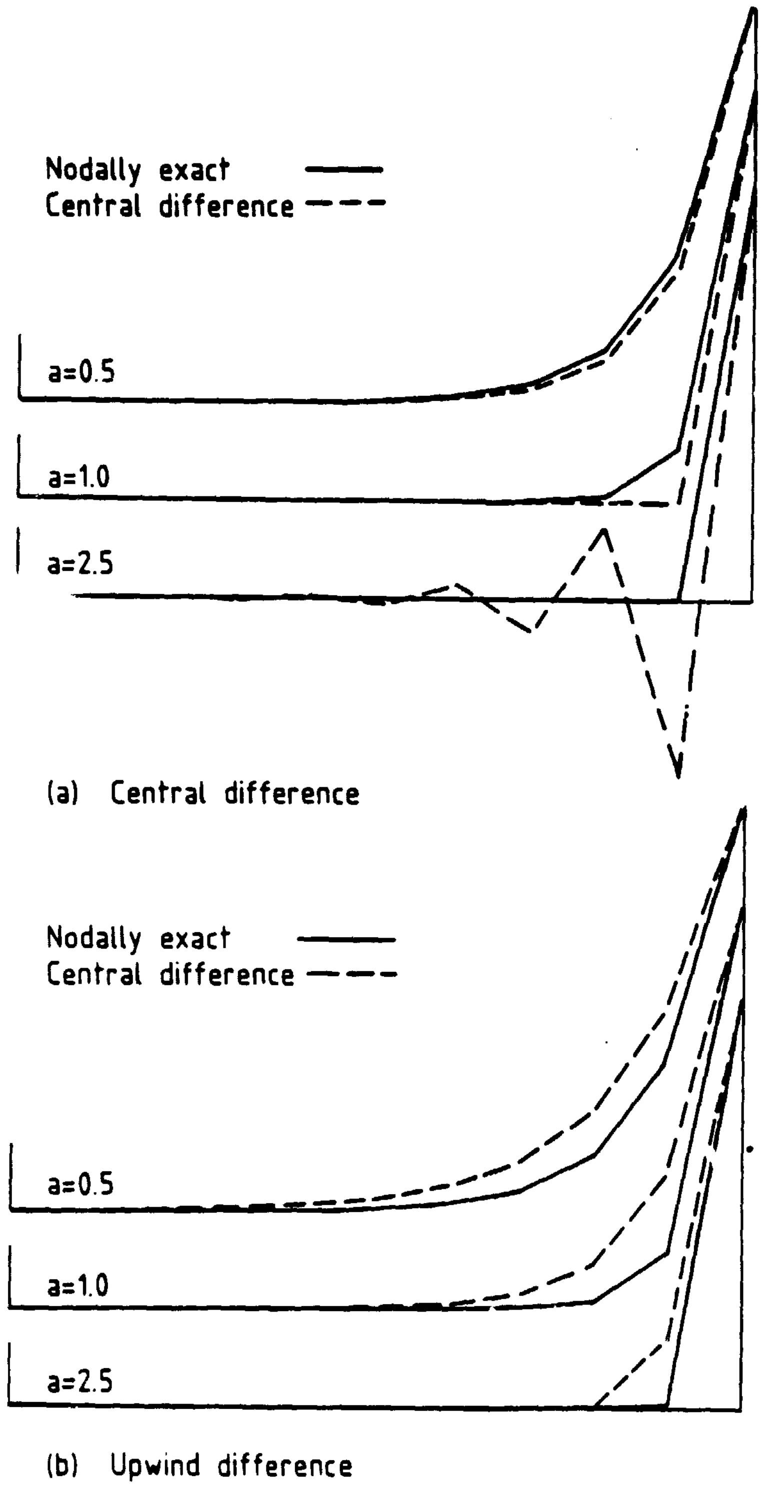

Figure 1. Steady advection-diffusion in one dimension:

$$u\frac{\partial T}{\partial x} = k\frac{\partial^2 T}{\partial x^2}, \quad a = \frac{uh}{2k}$$

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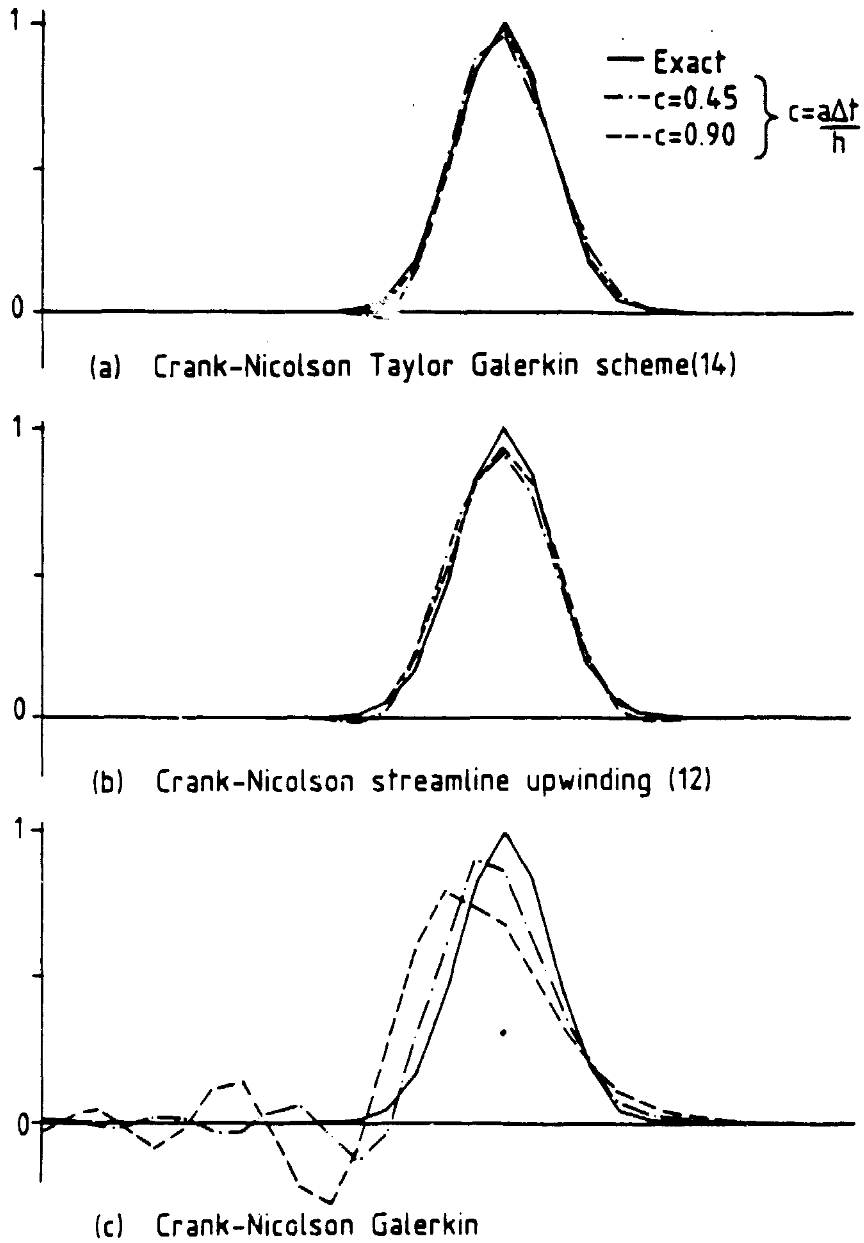

Figure 2. Solutions of $\frac{\partial T}{\partial t} = a \frac{\partial T}{\partial x}$.

	element-by-element	globally implicit
linear	$0(N^2)$	$0(N^3)$
non-linear	$0(N^2)$	$0(N^4)$

(a) Operation count comparison in two dimensions (per time step)

	element-by-element	globally implicit
linear	$0(N^3)$	$0(N^5)$
non-linear	$0(N^3)$	$0(N^7)$

(b) Operation count comparison in three dimensions (per time step)

Figure 3. A comparison of operation counts between the element-by-element algorithm of Hughes et al. (ref. 25), and the standard globally implicit methods for a finite element solution of a problem involving N nodes with a single degree of freedom per node.

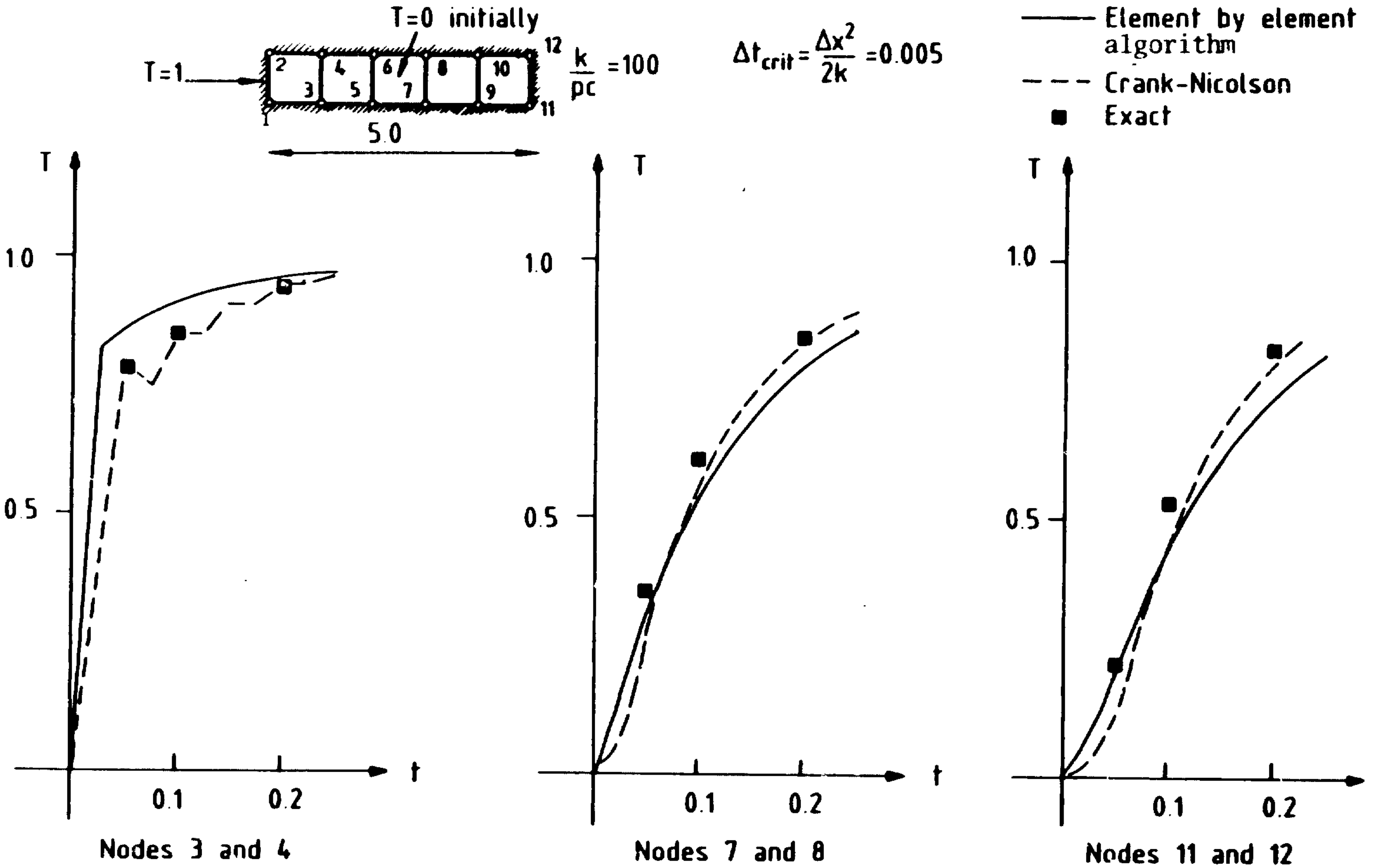

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Figure 4. Element-by-element algorithm used to solve the equation $\rho c\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}$ with time step $\Delta t = 0.025$.

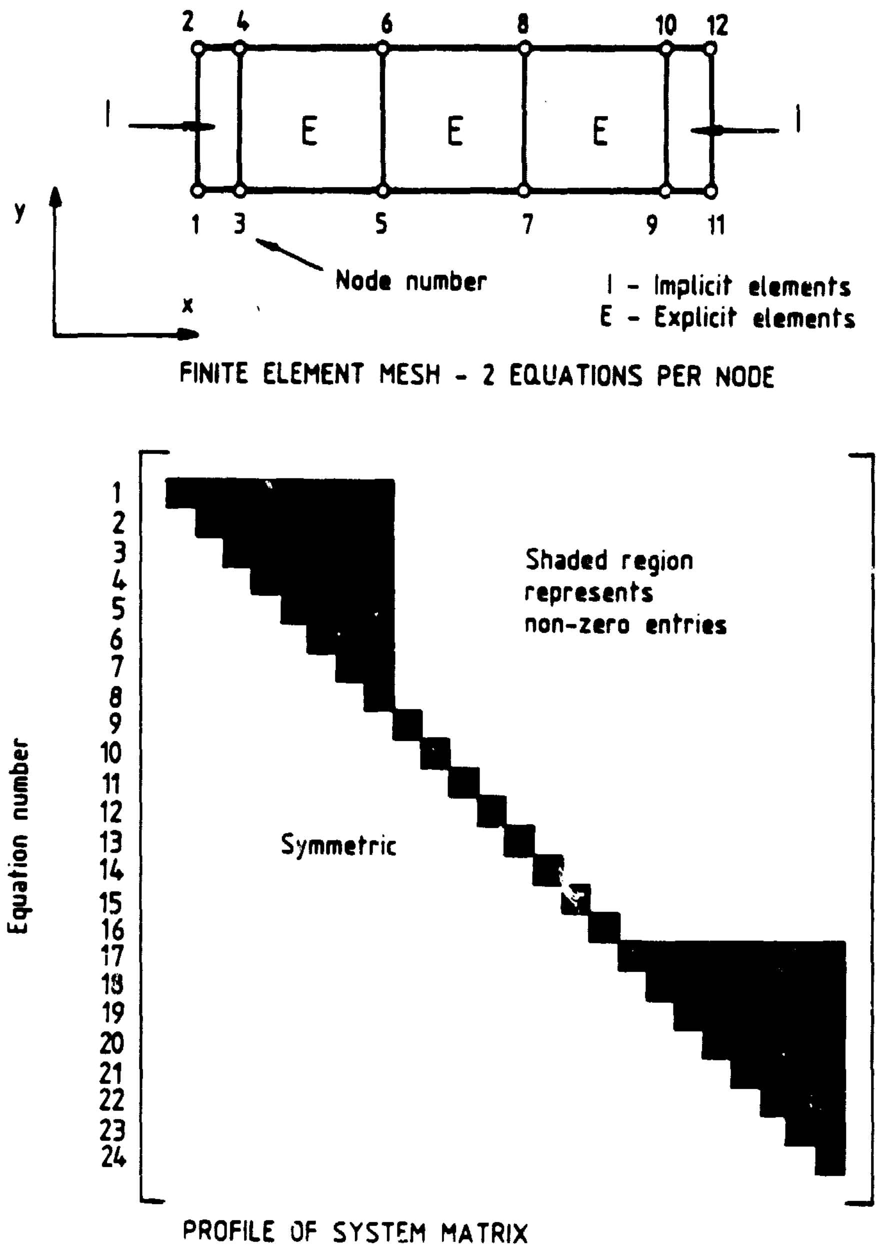

Figure 5. Two-dimensional implicit-explicit finite-element mesh and profile of system matrix (from Hughes and Liu, ref. 27).

ODU/Langley	Swansea
Stage 1	
Aug. 82 - June 83	
The production of a 2D unsteady code VISCOH2D designed to be as simple as possible and initially to be tested on 1D shock tube problems.	Evaluation of solution algorithms and identifying possible artificial viscosity models.
Stage 2	
June 83 - Sept. 83	
2D calculations using VISCOH2D - numerical comparisons.	
Stage 3	
Sept. 83 - June 84	
Implementation and testing of improved solution algorithms into VISCOH2D - numerical and experimental comparisons.	Evaluation of methods designed to improve code efficiency e.g. integration techniques.
Stage 4	
June 84 -	
Testing and running of the optimum 3D code VISCOH3D - numerical and experimental comparisons.	

Figure 6. Proposed stages in the development of a finite element based computer code for solving problems of three-dimensional high speed viscous compressible flows.